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NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 21: Seismic Response of Pile Foundations-1

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Introduction - Maheshwari & Watanabe (2009)

Proper design of pile-supported structures requires an adequate dynamic analysis of the pile foundations. Recognizing the importance of the soil-pile interaction, researchers have conducted extensive studies for analyses of pile groups. Development in pile-dynamics started about four decades ago. Initially proposed approaches were for one-dimensional linear analyses of single piles in the frequency domain. Due to extensive research and advancement in computational facilities, more sophisticated approaches have evolved over the time. Recently, researchers have proposed three-dimensional nonlinear dynamic analyses in the time domain for pile groups. However, yet relevance of simplified theories developed in sixties and seventies is not diminished. In fact, most of the later theories were just extension of the few basic theories developed earlier.

Introduction (Continued...)

For static loads on pile foundations, pioneer work has been done by Poulos (1968). In pile-dynamics, first widely known work is due to Penzien et al. (1964). In the broad sense, research was primarily focused on linear dynamic analysis. Substantial research efforts have been made on the linear analysis of single pile and pile groups in the frequency domain. This include work of many researchers such as Tajimi (1969), Novak (1974), Blaney et al. (1976), Nogami and Novak (1976), Novak and Nogami (1977), Kaynia and Kausel (1982), Gazetas (1984), Dobry and Gazetas (1988), Makris and Gazetas (1992), El Naggar and Novak (1996), Maheshwari and Watanabe (2006, 2009).



Introduction (Continued...)

Few researchers performed the time domain analyses. Matlock et al. (1978) developed a unit load transfer curve (p-y curves), for the time domain nonlinear analysis. Based on plane strain medium assumption in the frequency domain (Novak et al., 1978), Nogami and Konagai (1986 and 1988) have developed time domain analysis methods for axial and lateral response of single piles, respectively. Their analyses used concept of Winkler soil model. Later the time domain analyses using finite element concept is carried out by many researchers, for this one can refer the work done by Maheshwari et al. (2004, 2005).

Basic Concepts

During vibrations, the pile interacts with surrounding soil, modifying the stiffness of the pile and generating damping. For a soil-pile group system, two kinds of interaction effects are important. First is interaction between soil and pile, called soil-pile interaction. Second is interaction among piles through soil, i.e. pile-soil-pile interaction. An adequate analysis of a pile group should account for both of these interactions (Wolf, 1985).

Soil is a semi-infinite medium and a major problem in dynamic soil-structure interaction is modeling of the unbounded soil domain. For dynamic loading, structure (piles or superstructure) always interacts with surrounding soil and it is not adequate to analyze the structure independently. When seismic loading is applied to the soil region around the structure, then this region should be modeled along with the structure in such a way as to satisfy the radiation condition i.e. only waves can radiate outside the considered region and they cannot come inside this boundary. Thus this boundary represents an energy sink where only outgoing waves exist. Also it is necessary to enforce compatibility and force equilibrium condition between soil and pile along the pile surface for each pile considered.

Analysis of the Settlement of the Pile Groups, Poulos (1968)

For static analysis of piles, pioneer work has been done by Poulos (1968) considering both a single pile and a pile group. Both axial and lateral modes of vibration were considered. The theory is able to deal with end bearing as well as floating piles. Soil is assumed to be an ideal elastic, homogeneous, isotropic mass having constant elastic parameters. Though this idealization differs in many respects from reality but still it gives satisfactory results. Poulos theory have little significance in pile dynamics however, this provides a base for the development of various dynamic theories. For static case, Poulos defined few key factors like interaction factor for pile groups, which were subsequently defined for dynamic analysis in the same fashion. Also for parametric study, slenderness ratio, spacing between piles, relative rigidity between soil and pile were used by Poulos (1968), which were later used in dynamic theories too in a similar fashion.



Seismic analysis of Bridges on Long Piles, Penzien (1964)

Penzien et al. (1964) presented a nonlinear discrete dynamic model for piles passing through deep clay medium. These long piles were used to support a highway bridge. Horizontal mode of vibration is considered in the study. This model being a discrete model, allows for nonlinearity and account for gapping between soil and pile. However, it is very difficult to incorporate radiation damping and inertial effect of soil is accounted only approximately using lumped masses. Since radiation damping and inertial effect of soil are important parameters in a soil-structure interaction analysis, hence this model is not rational for dynamic analysis of pile group.

Dynamic Analysis of a Structure Embedded in an Elastic Stratum

Tajimi (1969)

Tajimi (1969) used a linear viscoelastic stratum of the Kelvin-Voight type to model the soil and did not consider the vertical response. This model is used only for dynamic analysis of a single pile resting on rigid bedrock. Viscous damping of soil stratum is taken into account. For the analysis of a single pile this theory was extensively used in Japan and described in detail as Tajimi's theory in later slides.

Soil-pile Interaction in Vertical and Horizontal Vibration

Nogami and Novak (1976), Novak and Nogami (1977)

Nogami and Novak (1976) developed a rigorous method for dynamic analysis of a single end bearing pile for axial vibration. Novak and Nogami (1977) extended the same for horizontal vibration. This theory is based on three-dimensional model and fully accounts for soil-pile interaction. Soil reaction is completely derived in this theory from basic principles. Hysteretic damping of soil is taken into account. This is most advance theory for a single pile for linear elastic analysis in the frequency domain and described in detail as Novak's 3D model.

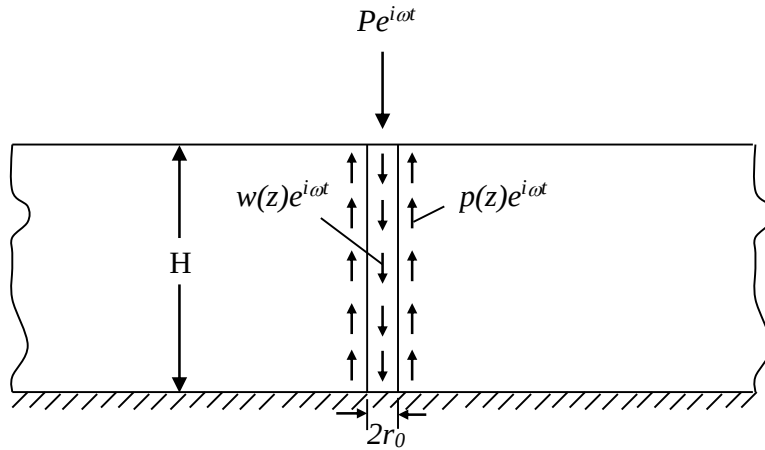
Plane Strain Soil Model (1978)

Situation of plane strain arises e.g. when a rigid cylinder extending to infinity in a semi-infinite medium undergoes uniform lateral displacements or rotates around its axis. Such model was also previously used by Novak (1974) but in that case soil reaction was taken that given by Baranov (1967). But in plane strain model, soil reaction is derived by approximating the solution derived by Nogami and Novak (1976).

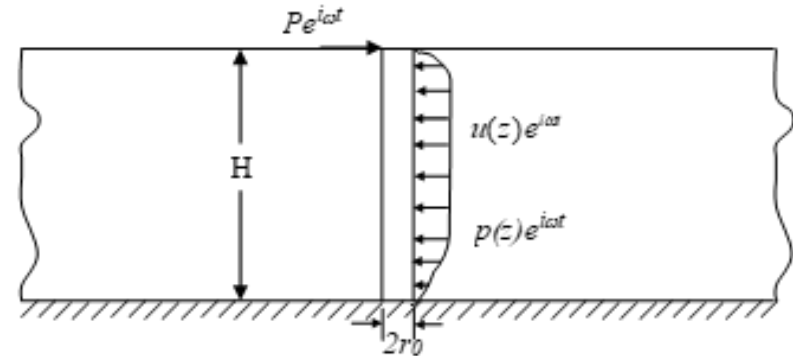
Theory is applicable for pile group also. It is assumed that response of the soil at one level is independent of at any other level and the piles are resting on bedrock. It can also be used for floating piles, if depth of the hard strata below the pile-tip is known. In the later case it is assumed that there is a virtual pile, made of soil, below the pile-tip up to the hard strata. Thus for floating piles, instead of piles only, pile-soil column is considered. Soil surrounding the columns is treated as a system of horizontal springs connected between the columns but uncoupled along the length of the column. Parameters of these springs are determined from plain strain condition.

Modeling for End-Bearing Single Soil-Pile System

Soil is considered homogeneous and assumes that piles are resting on bedrock thus end-bearing piles are considered. Fig. a and Fig. b show the soil-pile system for vertical and horizontal vibration, respectively.



(a)



(b)

Tajimi's Theory (Tajimi 1969): Horizontal Vibration

In general this theory is applicable for dynamic analysis of a structure embedded in an elastic stratum and vertically supported on the underlying bedrock. The analysis involved application of the three-dimensional wave propagation to the stratum overlying the bedrock.

Assumptions:

1. The model ground consists of an elastic stratum resting on the bedrock. A pile or pier of cylindrical shape resting on the bedrock is fully or partially embedded into the stratum.
2. The vertical displacement of the surface layer is neglected, because it will be less significant than the horizontal components.
3. The viscous damping of the stratum is considered for the shear wave velocity traveling along the depth.
4. The input acceleration is applied on the surface of the bedrock.

Tajimi's Theory (Tajimi 1969) (Continued...)

Formulation and Solution:

Since vertical displacement is neglected hence analysis requires solution of two wave equations in cylindrical coordinates (r, θ, z) . To separate the variables two potential functions are involved which reduces original equations of motion in the form of modified Bessel equation. By eliminating unknown constants with respect to boundary condition and expressing resistance of soil in terms of displacement, leads to complex soil stiffness of viscoelastic soil medium for horizontal vibration. This will be frequency dependent dynamic stiffness of soil and employs modified Bessel functions.

Once soil resistance at a particular frequency is known, solution of the pile-soil system is performed as a beam. By enforcing displacement compatibility and force equilibrium between soil and pile, the unknown constants are found. All formulation is in continuous form. Final solution is as follows:

Tajimi's Theory (Tajimi 1969) (Continued...)

Complex Stiffness of Soil:

Soil reaction in horizontal vibration, along the depth z , can be expressed as

$$p(z) = \pi\mu \sum_{n=1,3,\dots}^{\infty} \alpha_{hn} U_n \sin\left(\frac{n\pi z}{2H}\right) \quad (1a)$$

where U_n denotes the amplitude of the horizontal displacement in n th mode and dimensionless resistance factor (α_{hn}) for n^{th} mode at a particular frequency ω , is given by

$$\alpha_{hn} = \left(\frac{\pi}{2} \xi_n \frac{r_0}{H} \right)^2 \Omega_n \quad (1b)$$

Where,

$$\Omega_n = \frac{1}{\chi_n} \{4K_1(q_n r_0)K_1(s_n r_0) + s_n r_0 K_1(q_n r_0)K_0(s_n r_0) + q_n r_0 K_1(s_n r_0)K_0(q_n r_0)\} \quad (1c)$$

$$\chi_n = \{K_1(q_n r_0) + q_n r_0 K_0(q_n r_0)\} \{K_1(s_n r_0) + s_n r_0 K_0(s_n r_0)\} - K_1(q_n r_0)K_1(s_n r_0) \quad (1d)$$

Tajimi's Theory (Tajimi 1969) (Continued...)

$$\xi_n = \sqrt{\left\{ n^2 \left(1 + 2ih_g \frac{\omega}{\omega_g} \right) - \left(\frac{\omega}{\omega_g} \right)^2 \right\}} \quad (1e)$$

$$q_n = \frac{\xi_n \omega_g}{V_L} \quad \text{and} \quad s_n = \frac{\xi_n \omega_g}{V_S} \quad (1f)$$

where ω_g is lowest natural circular frequency of the surface layer and given by:

$$\omega_g = \frac{\pi}{2H} V_S; \quad V_S = \sqrt{\frac{\mu}{\rho}} \quad \text{and} \quad V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad (1g)$$

In above equations K_0 and K_1 are second modified Bessel functions of zero and first order, respectively. V_L and V_S are the propagation velocities of longitudinal and transverse waves, respectively. λ and μ are the Lamé's constants, and ρ is the mass density of soil medium. Further r_0 is the radius of the pile and H is thickness of soil stratum, which is equal to length of pile considered. The h_g denotes the critical damping ratio (viscous) for fundamental frequency of soil layer.

Tajimi's Theory (Tajimi 1969) (Continued...)

Complex Pile-Head Stiffness:

This can be defined as the force at the head of the pile due to unit displacement and can be written in complex form where real part represents spring constant while imaginary part damping. This can be expressed as:

$$K_p = EI \left(\frac{\pi}{2H} \right)^4 \frac{H}{2} [F(z, \omega)]^{-1} \quad (2a)$$

where dimensionless flexibility function F is given by

$$F(z, \omega) = \sum_{n=1,3,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^4 - \alpha_2 \frac{\rho_p}{\rho} \left(\frac{\omega}{\omega_g} \right)^2 + \alpha_2 \xi_n^2 \Omega_n} \sin \left(\frac{n\pi z}{2H} \right) \quad (2b)$$

Where,

$$\alpha_2 = \frac{\mu\pi}{EI} \left(\frac{2H}{\pi} \right)^4 \left(\frac{\pi_0}{2H} \right)^2 \quad (2c)$$

where E , I , ρ_p are the Young's modulus, moment of inertia, mass density of pile, respectively

References

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Thank You

